

A Re-evaluation of Heat Transfer Associated with Drawing of Polymers

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Synopsis

The temperature rise due to heat generation associated with drawing of polymer is re-evaluated and rederived starting from the first law of thermodynamics. The errors of both formulations and sample calculations made in a previous investigation were corrected. The predicted efficiency or the fraction of mechanical work converted into heat during necking based on the present formulations is found to be reasonable.

INTRODUCTION

The importance of heat generation during the deformation of polymers has been well recognized for sometime. As a result, numerous investigations¹⁻¹⁰ have been performed on this topic. A recent study of Maher et al.¹¹ has demonstrated that the heat is generated in the necking region when a polymer specimen is subjected to tension by the cold-drawing process. The authors have shown that the inverse of temperature rise, $1/\Delta T$, during the drawing process is a linear function of the inverse of neck velocity, $1/v_c$. This study has been further extended by Warner¹² to model the heat transfer of a fiber subjected to the nonuniform and uniform deformation plus crystallization. Unfortunately, this article contains errors in both theoretical formulations and numerical calculations. Since the article under consideration represents an important heat transfer process associated with drawing of polymers, it is believed that the problem warrants further investigation and rectification.

In this work, the same problem attempted by Warner is re-examined and resolved. The corrected formulations and sample calculations for temperature rise during the drawing of PET (polyethylene terephthalate) are presented. For ease of comparison, the same notations and format of Ref. 12 are employed in this work.

FORMULATION

As with Warner,¹² the deformation process during the drawing of polymers is divided into two stages, namely, neck propagation or drawing to the natural draw ratio and uniform deformation accompanied by crystallization. In the first stage, the necking takes place in a very short distance. The temperature

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rise in this stage can be derived from the first law of thermodynamics¹³

$$\dot{W} = \dot{Q} + dU/dt \quad (1)$$

where \dot{W} is the rate of work applied to the system, \dot{Q} is the heat transfer rate from the system, and dU/dt is the rate change of internal energy of the system. This equation can be approximated by

$$\dot{W}\Delta t = \dot{Q}\Delta t + \Delta U \quad (2)$$

Applying the lumped-heat capacity analysis and assuming that the initial temperature and the ambient temperature are equal, one can write

$$\alpha L g v_c \Delta t = h A \Delta T \Delta t + V \rho c_p \Delta T \quad (3)$$

where L is draw tension, v_c is neck velocity, Δt is the time interval, α is efficiency or the fraction of mechanical work which converts into heat energy; A is deformation surface area, V is the volume of the necked material, h is the convective heat transfer coefficient, ρ is density, and c_p is specific heat and $g = 9.8$ N/kg. Equation (3) can be rewritten as

$$1/\Delta T = hA/(\alpha L g v_c) + (\rho c_p V/\Delta t)/(\alpha L g v_c) \quad (4)$$

The above formula disagrees with Warner's equation (1) which states

$$1/\Delta T = -hA/(\alpha L g v_c) + ab\rho c_p/[(D-1)Lg\alpha]$$

where a and b are the width and thickness of the specimen respectively. The minus sign on the above expression does not agree with physics. Furthermore, it seems to be more appropriate to represent the volume of the fiber by

$$V = \pi r_a^2 v_c \Delta t \quad (5)$$

instead of Eq. (5) of Ref. 11, which is repeated below,

$$V = \pi r_b^2 v_c \Delta t / (D - 1) \quad (6)$$

(r_a and r_b are radii after and before drawing, respectively, and D is draw ratio $\pi = 3.14159\dots$) since the temperature rise takes place mainly in the neck and the necking distance is very short. It is believed that Eq. (6) tends to overestimate the actual volume. Since the first term on the right-hand side of Eq. (4) is relatively small, one can write from Eq. (4)

$$1/\Delta T \doteq (\rho c_p V/\Delta t)/(\alpha L g v_c) \quad (4a)$$

For a given measured ΔT and thermal properties, an overpredicted volume, V [from Eq. (6)] would yield a larger value of α from Eq. (4a). This is probably why Maher et al.¹¹ found $\alpha > 1$ for certain cases. Substituting Eq. (5) into Eq.

(4) gives the expression for temperature rise in stage one.

$$1/\Delta T = hA/(\alpha Lg v_c) + \pi r_a^2 \rho c_p / (\alpha Lg) \quad (7)$$

In the second stage, a uniform deformation accompanying crystallization occurs. Under this condition, the energy associated with the stress-induced crystallization should be included in the energy balance equation, in other words, an additional term is required in Eq. (3).

$$\alpha Lg v_c \Delta t + x \Delta H_c V \rho = hA \Delta T \Delta t + \rho c_p V \Delta T \quad (8)$$

where x is crystallinity and ΔH_c represents the heat release per unit mass of crystal during crystallization. Equation (8) can be rewritten as

$$1/\Delta T = hA/(\alpha Lg v_c + x \Delta H_c V \rho / \Delta t) = (\rho c_p V / \Delta t) / (\alpha Lg v_c + x \Delta H_c \rho V / \Delta t) \quad (9)$$

This expression is completely different from Warner's equation (2) which is repeated below

$$1/\Delta T = hA/(\alpha Lg v_c) + \pi r_a^2 \rho c_p / [(D-1)Lg\alpha] + c_p(x \Delta H_c) \quad (10)$$

Equation (10) implies that $\Delta T = 0$ if $x = 0$, which is not true physically.

Since the deformation process in the second stage is relatively long and the deformation is uniform, the volume and surface area can be more accurately computed from:

$$V = \pi(r_a^2 + r_b^2)v_c \Delta t / 2 \quad (11)$$

$$A = \pi(r_a + r_b)\ell \quad (12)$$

SAMPLE CALCULATIONS

The same example as employed by Warner¹² is employed here; drawing conducted between a feed roll and a draw roll. The diameter before necking deformation is 40 μm ; after necking and before uniform deformation is 27 μm , and after uniform deformation is 19 μm . The corresponding denier are 16, 6.7, and 3.4, respectively, which in turn yield the diameters of 41×10^{-5} m, 26.4×10^{-6} m, and 18.8×10^{-6} m, respectively, from the following definitions of denier:

$$\text{denier} = 9,000 D^2 \rho \pi / 4 \quad (13)$$

Stage I

The filament denier changes from 16 to 6.7 during its deformation in this stage. The initial temperature of the fiber and the ambient temperature are the same and equal to 293°K. The following data were given by Warner: $v_c = 0.2$ m/s, $L = 8.8 \times 10^{-3}$ kg, $A = 2 \times 10^{-9}$ m², $c_p = 1.12$ kJ/kg, $\rho =$

$1.34 \times 10^3 \text{ kg/m}^3$. The correct heat transfer coefficient, h is $19 \times 10^{-3}(\Delta T)^{1/4}$ $\text{kJ/m}^2\text{-s-K}$ based on the present calculation in the Appendix. The value of α should be obtained from the experimental data which is not available for PET at present. However, it can be estimated from Eq. (3) in light of experimental data by Maher et al.¹¹ and Thompson for ΔT which list 60°K and 58°K , respectively. Substituting the above data into Eq. (3) gives $\alpha = 0.563$ and 0.544 , respectively, which are considered to be quite reasonable. Also from $A = 2\pi r_a \ell$, one obtains $\ell = 2.45 \times 10^{-5} \text{ m}$ for this example.

Stage II

The filament denier changes from 6.7 to 3.4 during the deformation of stage II and the initial temperature of the fiber and the ambient air are both at 353°K . The following data was obtained from Warner: $d_b = 26.2 \times 10^{-6} \text{ m}$, $d_a = 18.8 \times 10^{-6} \text{ m}$, $v_c = 0.3 \text{ m/s}$, $L = 1.5 \times 10^{-3} \text{ kg}$, $\ell = 1 \text{ m}$, $c_p = 1.37 \text{ kJ/kg-K}$, $x = 0.4$ and $\rho = 1360 \text{ kg/m}^3$. However, the corrected values for heat of fusion (same as heat of crystallization numerically¹⁴) and heat transfer coefficient should read $\Delta H_c = 133 \text{ kJ/kg}$ (instead of 13 kJ/kg) and $h = 18.62 \times 10^{-3}(\Delta T)^{1/4} \text{ kJ/m}^2\text{-s K}$. The above correct values are obtained, respectively, from Brandrup and Immergut,¹⁵ and the present calculation from the Appendix. Furthermore, the volume and deformation area are computed from

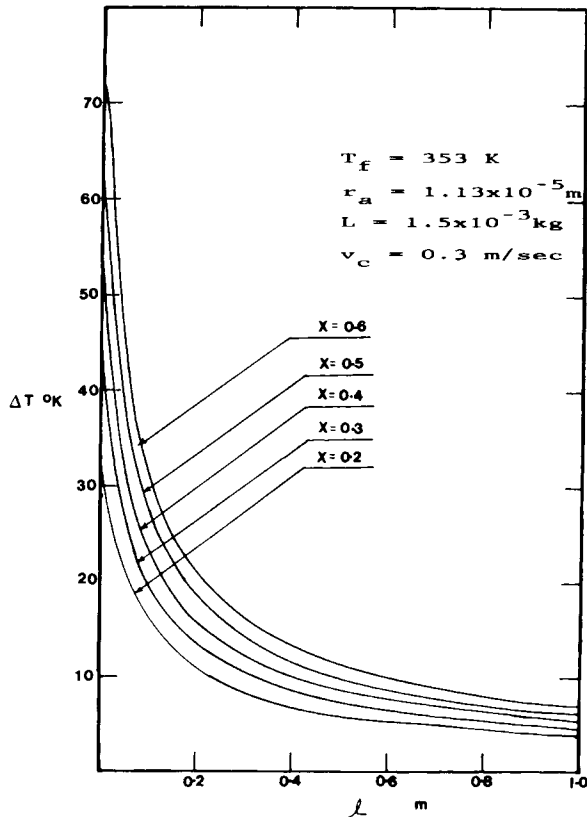


Fig. 1. Temperature rise vs. uniform deformation zone length with $\alpha = 0.68$.

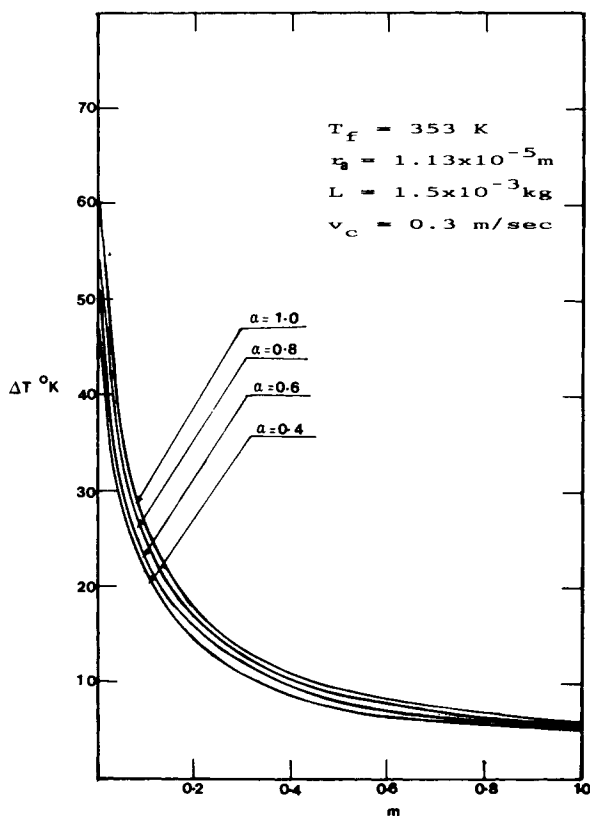


Fig. 2. Temperature rise vs. uniform deformation zone length with $x = 0.4$.

Eqs. (11) and (12), respectively, and are given by $V/\Delta t = 1.225 \times 10^{-10} \text{ m}^3/\text{s}$ and $A = 7.069 \times 10^{-5} \text{ m}^2$. Letting $\alpha = 0.68$ and substituting the above data into Eq. (8) yields

$$\Delta T = 51.97 / (5.769(\Delta T)^{0.25} + 1) \quad (17)$$

Solving for ΔT iteratively gives $\Delta T = 5.33^\circ\text{K}$.

If $\ell = 0.1 \text{ m}$ then

$$\Delta T = 51.97 / (0.577(\Delta T)^{0.25} + 1) \quad (18)$$

which yields $\Delta T = 23^\circ\text{K}$. If $\ell = 0$, in the adiabatic or the worst case, $\Delta T = 52^\circ\text{K}$. The variations of temperature rise at different values of x and α with respect to the length of drawing zone are shown in Figures 1 and 2, respectively, which graphically represent the formula $\Delta T = (19.32\alpha + 97.08x) / (5.769\ell(\Delta T)^{0.25} + 1)$.

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APPENDIX

Calculation of Heat Transfer Coefficient

Since v_c is relatively small, the free convection formulation is used to compute the heat transfer from the cylindrical fiber to the ambient. From Bird et al.¹⁶

$$\text{Nu} = 0.525(\text{GrPr})^{0.35} \quad (\text{A-1})$$

Substituting the definitions of Nusselt number, Grashof number, and Prandtl number into the above expression gives

$$h = 0.525[g \Delta T / (2r \cdot T_\infty)]^{1/4} [c_p \rho^2 k^3 \mu]^{1/4} \quad (\text{A-2})$$

Numerical computations reveal that the property group inside the second bracket is almost constant for air ($6 \times 10^{-3} \text{ kJ/s-K-m}^2$) within the temperature range of 273°K to 383°K .

In the first stage of deformation, $r = 1.32 \times 10^{-5} \text{ m}$, $T_\infty = 293^\circ\text{K}$. In the second stage of deformation, $r = 1.113 \times 10^{-5} \text{ m}$, $T_\infty = 353^\circ\text{K}$.

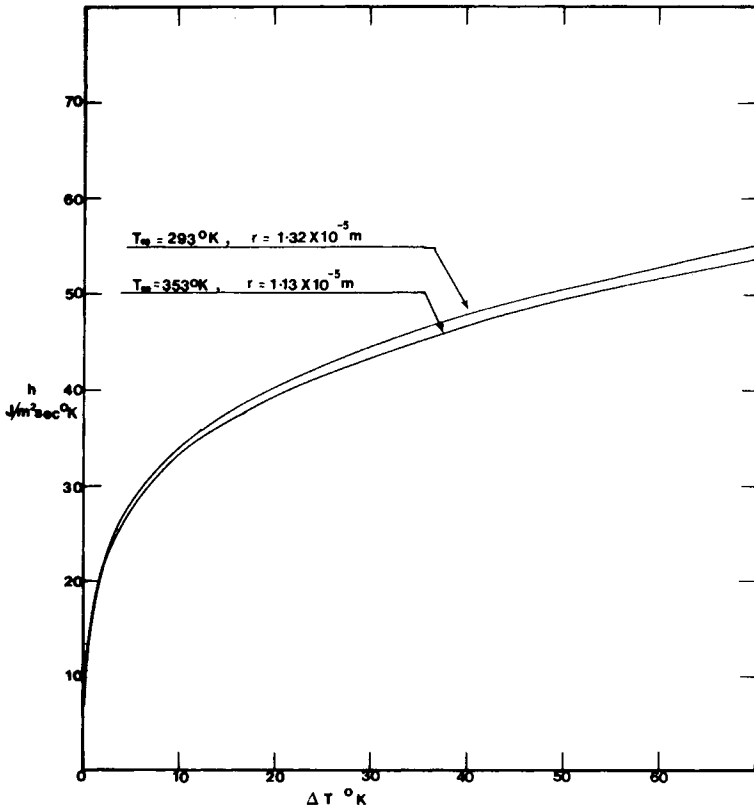


Fig. 3. The free convective heat transfer coefficient from a cylindrical fiber to air vs. temperature difference.

Substituting the above values into Eq. (A-2) gives $h = 19 \times 10^{-3}(\Delta T)^{0.25}$ kJ/m²-s-K for the first stage and $h = 18.62 \times 10^{-3}(\Delta T)^{0.25}$ kJ/m²-s-K for the second stage. Figure 3 shows the free convective heat transfer from a cylindrical fiber to air as functions of temperature difference for two different situations.

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